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Book Review: *Fractals in Science*

Fractals in Science. Armin Bunde and Shlomo Havlin, eds., Springer-Verlag, Berlin, 1994.

These nine essays provide succinct yet mostly accessible summaries of research at the frontier in fractal mathematics as applied to medicine, biology, ecology, polymers, condensed matter physics, and surface science. In these fields scaling concepts are particularly effective in quantifying and describing inherently complex phenomena. This second volume of a series highlighting applications of fractals in science continues the practice introduced in the first volume, *Fractals and Disordered Systems* (A. Bunde and S. Havlin, Springer-Verlag, Berlin, 1991) of providing an introductory chapter by the editors, some background material and literature citations within each chapter, plus relevant cross-references to other chapters in this and the first volume of the series. Notation for the various fractal dimensions is consistent from chapter to chapter and also with that found in the first volume.

The chapters and authors are:

Chapter 1: A Brief Introduction to Fractal Geometry, A. Bunde and S. Havlin

Chapter 2: Fractal and Self-Organized Critically, P. Bak and M. Creutz

Chapter 3: Fractals in Biology and Medicine, S. V. Buldyrev, A. L. Goldberger, S. Havlin, C.-K. Peng, and H. E. Stanley

Chapter 4: Self-Affine Interfaces, J. Kertesz and T. Vicsek

Chapter 5: A Primer of Random Walkology, G. H. Weiss

Chapter 6: Polymers, Catalysts, M. Daud

Chapter 7: Kinetics and Spatial Organization of Competitive Reactions, S. Redner and F. Leyvraz

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Chapter 8: Fractals Analysis in Heterogeneous Chemistry, D. Avnir, R. Gutfraind, and D. Farin

Chapter 9: Computer Exploration of Fractals, Chaos and Cooperativity, D. C. Rapaport and M. Meyer

Chapter 1 reviews fundamental notation and terminology and describes several well-known deterministic fractals and the three random-fractal models of importance in most applications: random walks, percolation, and diffusion-limited aggregation (DLA). Pertinent features of self-affine (rough) surfaces and time series are defined and discussed as well. This chapter will not serve to introduce the reader to the field of fractals, but it will enable a somewhat knowledgeable reader to get up to speed on the subject.

Chapter 2 proposes an explanation for the ubiquity of fractal structures in nature: Friction in real dynamical systems causes large, multi-dimensional systems to settle near minimally stable states, far from the analog of thermal equilibrium. A sandpile is an example cited which accumulates added grains (because of friction) until it reaches a critical slope, after which more grains can trigger avalanches of any size (i.e., no size scale). "Self-organized criticality" (SOC) is the term applied to this tendency of large dissipative systems to be attracted to a critical stationary state, and it is suggested as a mechanism related to earthquakes, turbulence, and $1/f$ noise, although it appears that real sandpiles do not exhibit SOC.

Chapter 3 describes a number of familiar and new topics relating to fractal applications in biology and medicine. Multiple branched airways of the lung, neural nerve geometries, and viscous fingering at gastric glands are perhaps familiar, but correlations in the distribution of DNA sequences and the possible role of long-range temporal correlations in the pattern of heart beats are somewhat less so. This reviewer found the proposed "crumpled globule" conformation of DNA particularly interesting and thought-provoking, perhaps since this type of collapsed polymer coil has only very recently been observed in the case of synthetic macromolecules.

Chapter 4 surveys on-going efforts to explain the structure and dynamics of rough surfaces, with particular emphasis on a nonlinear (KPZ) diffusion model for fluctuations in surface-height displacements. While the addition of correlated and uncorrelated noise terms of KPZ can reproduce important features of real surfaces, much experimental and theoretical work remains to be done. The thought that the KPZ model may not be the best starting point for a physical analysis can enter a reader's mind.

In Chapter 5 Weiss presents a concise and instructive overview of results for homogeneous random walks, results readily obtained using basic properties of generating functions. The continuous-time random walk (CTRW) generalization arises naturally as a phenomenological model for structural and energetic disorder. The latter half of the chapter treats random walks in disordered media which are most relevant to this volume. The relatively undeveloped state and many unanswered questions in the theory of random walks in disordered media are abundantly clear from the exposition.

An excellent review of modern polymer physics is found in Chapter 6, with a timely discussion of chain conformations near surfaces, as well as the increasingly lively and important topic (after a 40-year lull) of gels and gelation.

Chapter 7 reviews kinetic and spatial organization effects of two-species annihilation which occurs when particles from two distinct species A and B react when they get sufficiently close. If the reaction times are fast compared with collision rates (i.e., the diffusion-limited case), domains rich in A and B can grow from an initially uniform system as the annihilation process induces an effective "repulsion" between A's and B's. Domain sizes, interparticle distributions, and interfacial domain densities and thicknesses have been determined for the case of initially uniform densities of A and B. For the case (relevant to catalysis) when the A and B particles are immobile but are replenished by steady injection, exact solutions have been found by mapping the system to the kinetic Ising model. While mathematical results are emphasized in the chapter, one expects that the complex mosaic structures predicted by theory may soon be observed in laboratory chemical reactions.

Chemical implications of complex geometries and fractal heterogeneous surfaces for pharmacology, catalysis, and chromatography are discussed in Chapter 8. An empirical fractal reaction dimension d_e is computed from experimental rates of drug dissolution and catalytic conversions. The reaction dimension may be less than, equal to, or greater than the fractal dimension of the surface, depending on the nature of the spatial irregularities on the surface, or the sizes and energetics of the reactants.

Chapter 9 describes a collection of software demonstrations provided on a diskette that accompanies the volume. The goal of the demonstrations is to allow the reader to experience a measure of the visual fascination attached to fractal images, especially those which one grows oneself. This reviewer has an old Mac II which could not read the highdensity diskette, but access to a II SI allowed it to be read and then transferred. The demos are instructive, and a worthwhile supplement to the other contributions. If a second edition were contemplated, it is suggested that specific demos be discussed in each chapter, and that a more sophisticated graphical display be made available.

More exposition and background overall would have been desirable, especially for the highly mathematical chapters on surfaces and kinetics. Perhaps a rubric of standard questions for each author could have been used:

(a) Why is this subject important?

(b) Historical background: What was known before the fractal models? (Many of these topics have been around for over 50 years.)

(c) What was the status 10 and/or 15 years ago?

(d) What issues have been resolved recently (using fractals, or otherwise), and what are the remaining outstanding problems?

(e) What impact have fractal models had on the field?

The answers to the above are obvious to the experts (perhaps), but the chapters are intended for nonexperts, and possibly non-fractal experts. If the chapters were refereed by knowledgeable, non-fractal reviewers, readability and accessibility may have been improved.

Notwithstanding stylistic shortcomings, the ease with which fractal methods foster interdisciplinary research and forge direct links between basic theory and practical applications is convincingly documented in this two-volume series. Ironically, in an era when influences tend to fragment the uses and canons of science, fractal models and mathematics provide a welcome unifying motif. Captivating exposition will probably have to await maturation of the field, but in the interim this series provides valuable access to an important field.

John Bendler
Department of Chemistry
South Dakota School of Mines and Technology
Rapid City, South Dakota 57701